Deterministic and stochastic buckling analysis for imperfection sensitive stiffened cylinders

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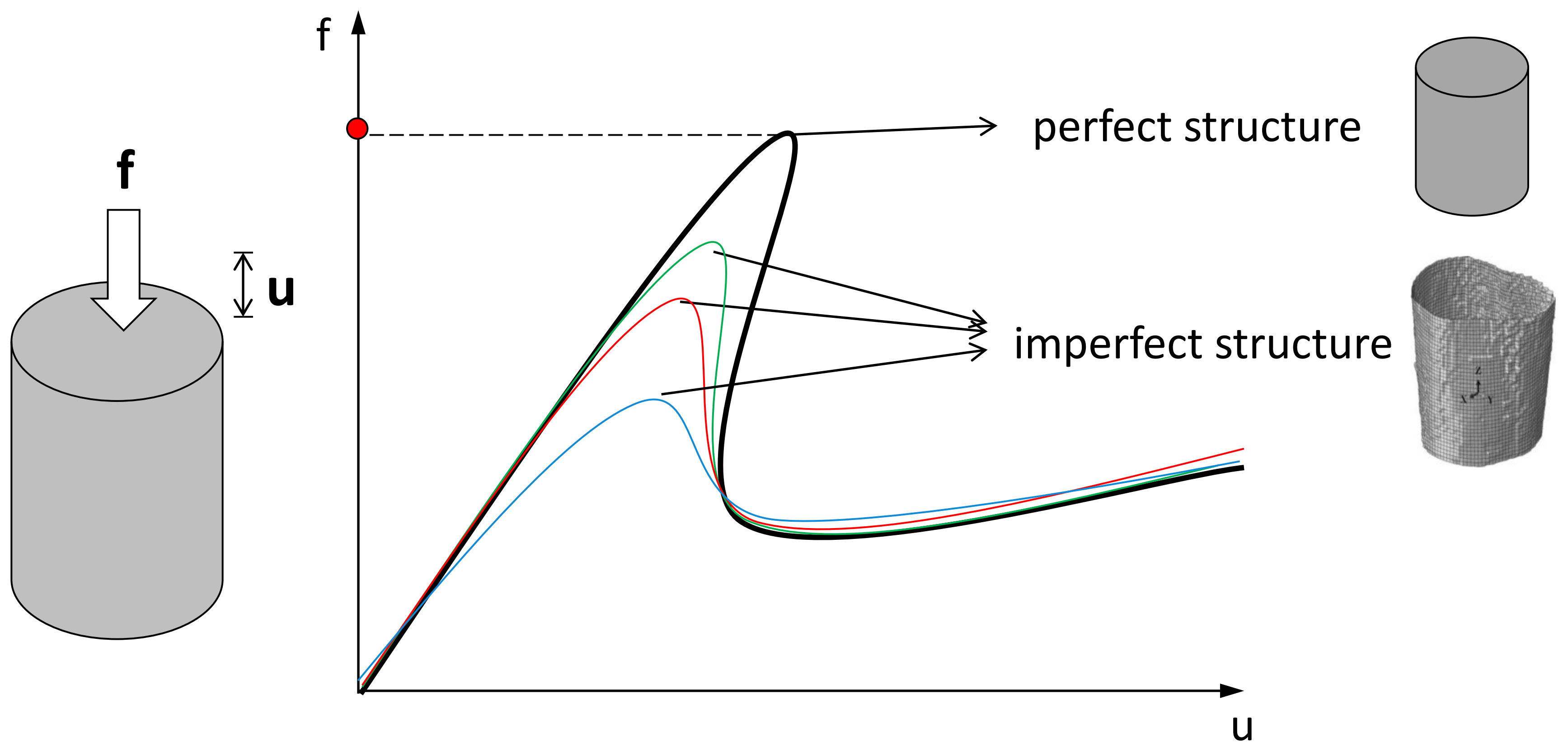
outline

- design concept with knock-down factors
- deterministic analysis
- stochastic analysis
- combined knock-down factor
- conclusions

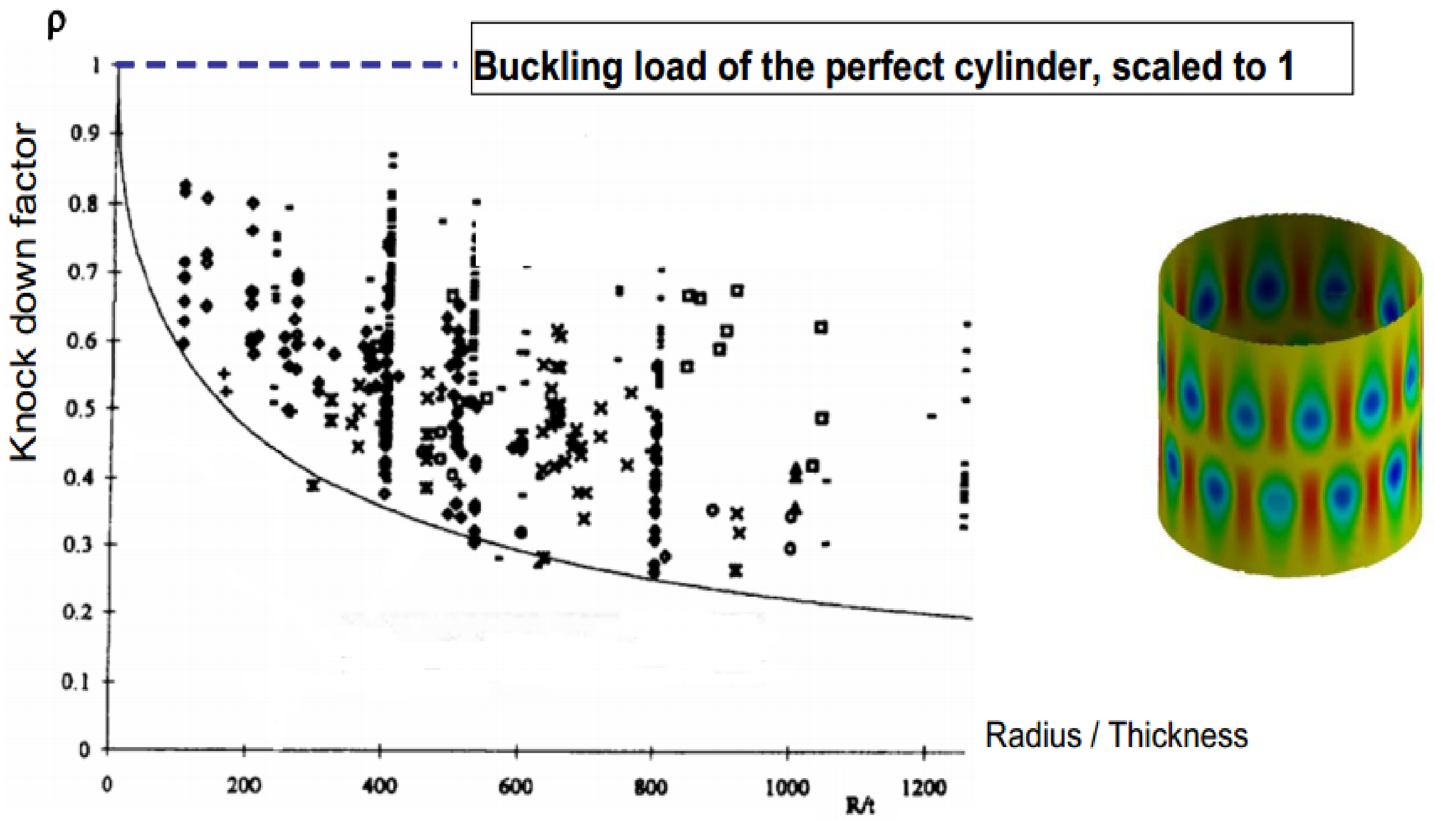
concept of knock-down factors – **introduction**



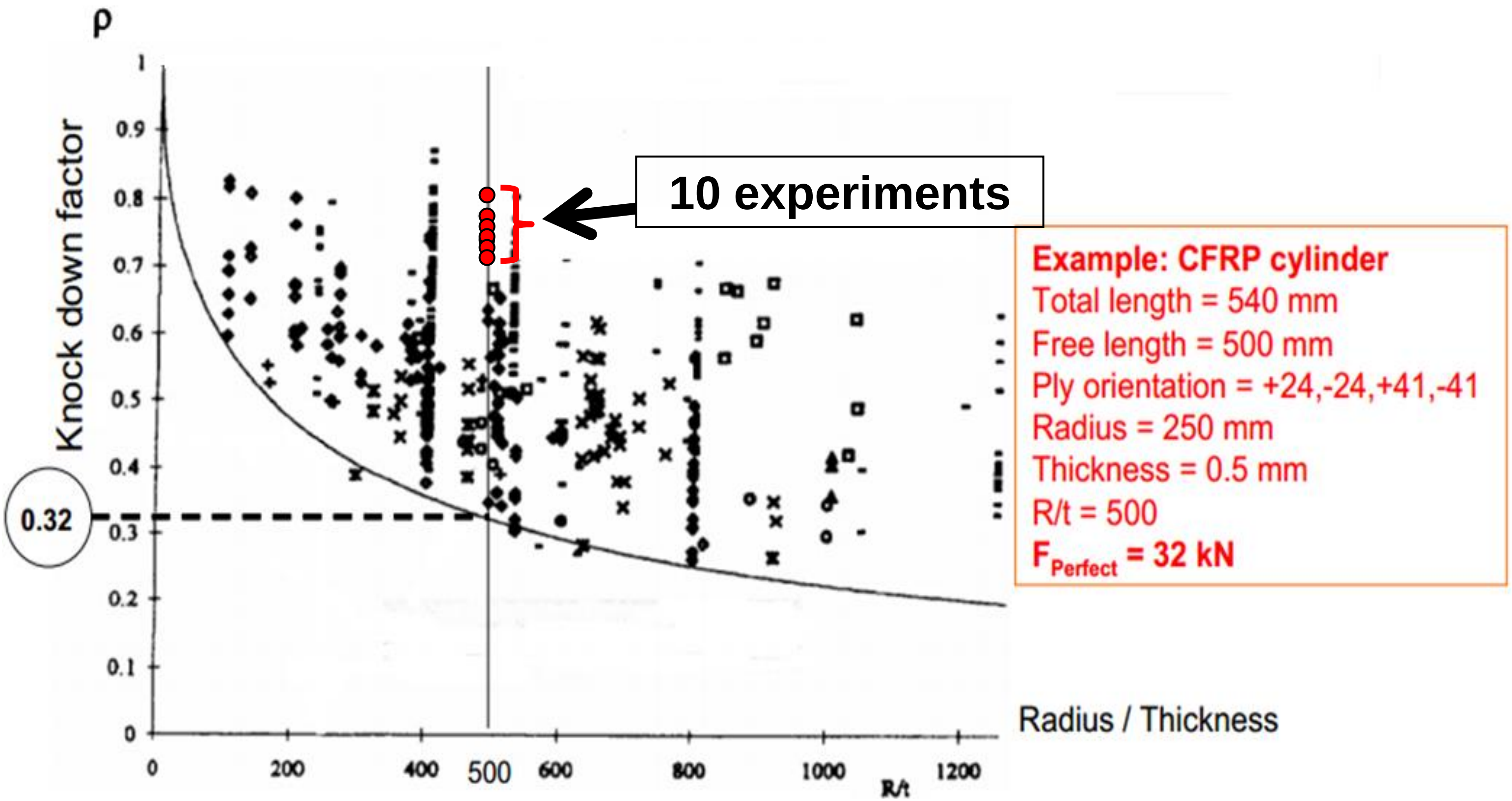
$$F_{design} = F_{perfect} \times k$$

k<1 knock-down factor

- standard design approch based on NASA SP-8007 (1968)
- provides lower-bound curves from experimental data



- experimental testing & numerical prediction improved
- SP-8007 seems to be too conservative



CFRP – carbon fibre reinforced polymer

less conservative design approach proposed, based on numerical simulation results

old: $$F_{design} = F_{perfect} \times k_{nasa}$$

$\Downarrow$

new: $$F_{design} = F_{perfect} \times k_1 \times k_2$$

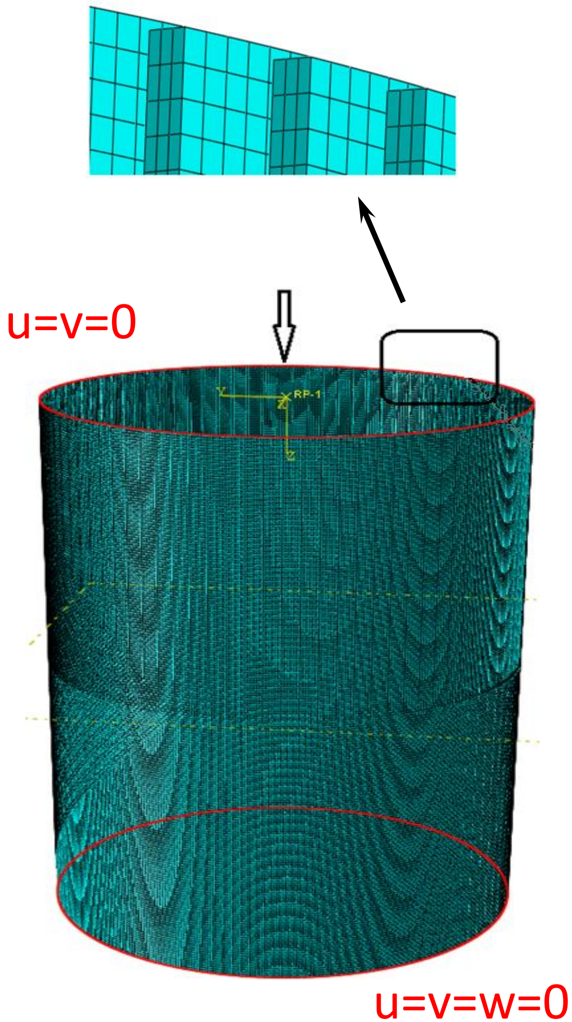
k_1 considers **geometric imperfection** using deterministic methods

k_2 considers **other imperfections** using stochastic methods the **new design concept** was tested exemplarily with two stiffened test cylinders

id	material	cylinder		skin	stiffener			NASA SP8007 knock-down factor	Test ?
	E, μ	*radius*	*height*	*thickness*	*thickness*	*height*	*number*		
A	70000, 0.34	400	1000	0.8	0.8	5.2	**90**	**0.4616**	?
B	70000, 0.34	400	1000	0.55	0.55	5.2	**126**	**0.4387**	YES

two different **numerical models** were used

- stringer shell model
- smeared shell model

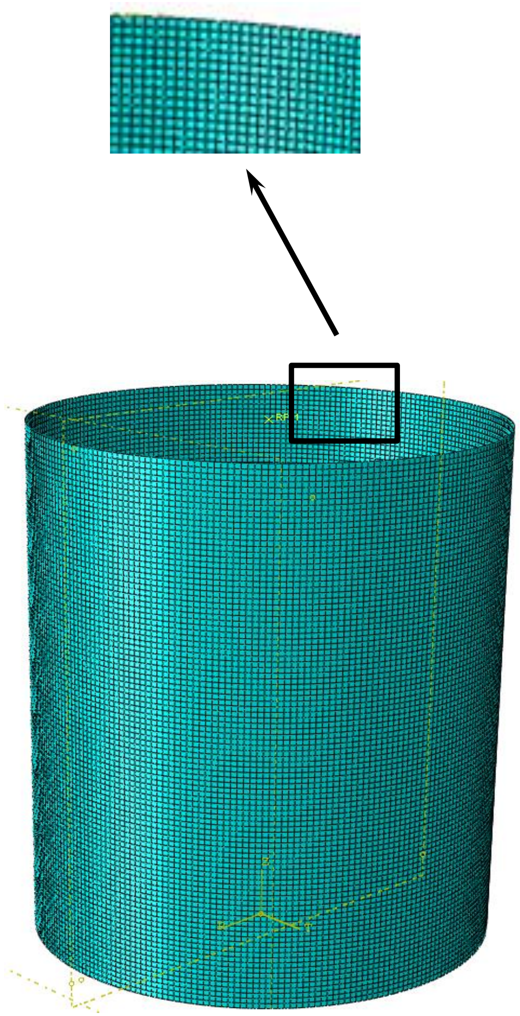


- explicitly modeled shell stringers
- 174960 S4R shell elements (Abaqus)
- S4R: reduced integration to avoid locking
- hourglass modes exist

discretization

- axial directions — 216 elements
- between two stringers — 6 elements
- stiffener height — 3 elements

buckling analysis – **smeared shell** model



- no modeled shell stringers
- 25100 S4R shell elements (Abaqus)

- less elements (factor 7)
- consideration of measured geometric imperfections of unstiffened cylinders

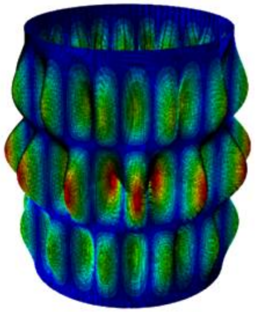
$$K = \begin{bmatrix} 73747.59668 & 21528.72 & 0 & 31283.4956 & 0 & 0 \\ 21528.72 & 63319.7648 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20895.5224 & 0 & 0 & 0 \\ 31283.4956 & 0 & 0 & 120724.922 & 1148.1984 & 0 \\ 0 & 0 & 0 & 1148.1984 & 3377.05412 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1321.9469 \end{bmatrix}$$

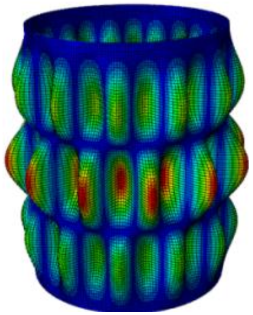
buckling analysis – **comparison model A**

- number of stiffeners 90
- thickness skin/stiffener 0.8 mm

model type	*linear* buckling load $F_{perfect}$
stringer model (174960 elements)	**205.92 kN**
smeared model (25100 elements)	**203.27 kN** *(rel. dev 1.29%)*

first buckling mode **stringer model**





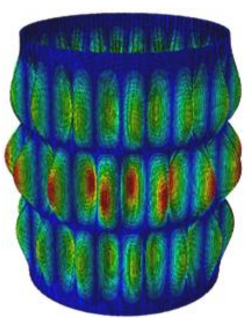
first buckling mode **smeared model**

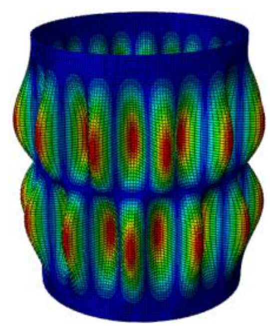
buckling analysis – **comparison model B**

- number of stiffeners 126
- thickness skin/stiffener 0.55 mm

model type	***linear*** **buckling load** $F_{perfect}$
stringer model (174960 elements)	**103.09 kN**
smeared model (25100 elements)	**103.76 kN** *(rel. dev 0.65%)*

first buckling mode **stringer model**





first buckling mode **smeared model**

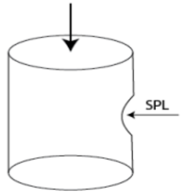
$$F_{design} = F_{perfect} \times k_1 \times k_2$$

k_1 considers **geometric imperfection** using deterministic methods

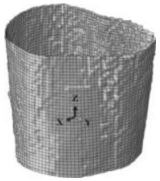
k_2 considers **other imperfections** using stochastic methods

methods used to model geometric imperfections

- single perturbation load approach (SPLA) applied to the **stringer model**

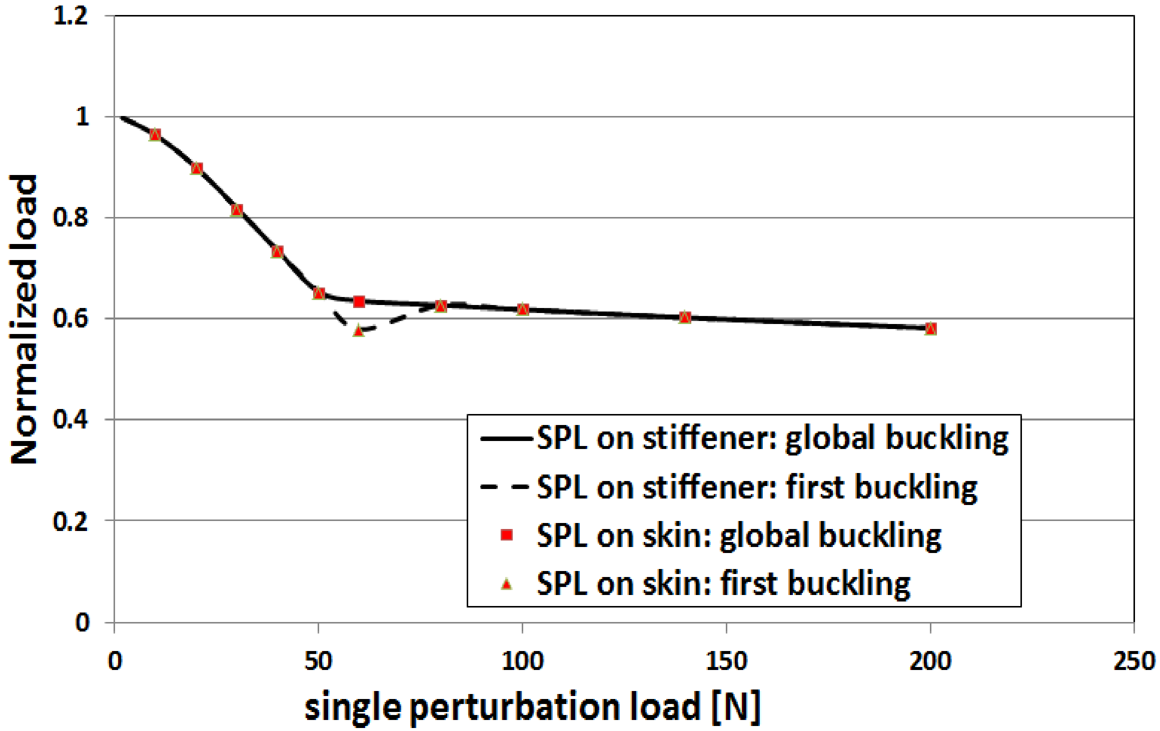


- modeling of measured imperfections (Z15, Z17, Z20) applied to the **smeared model**



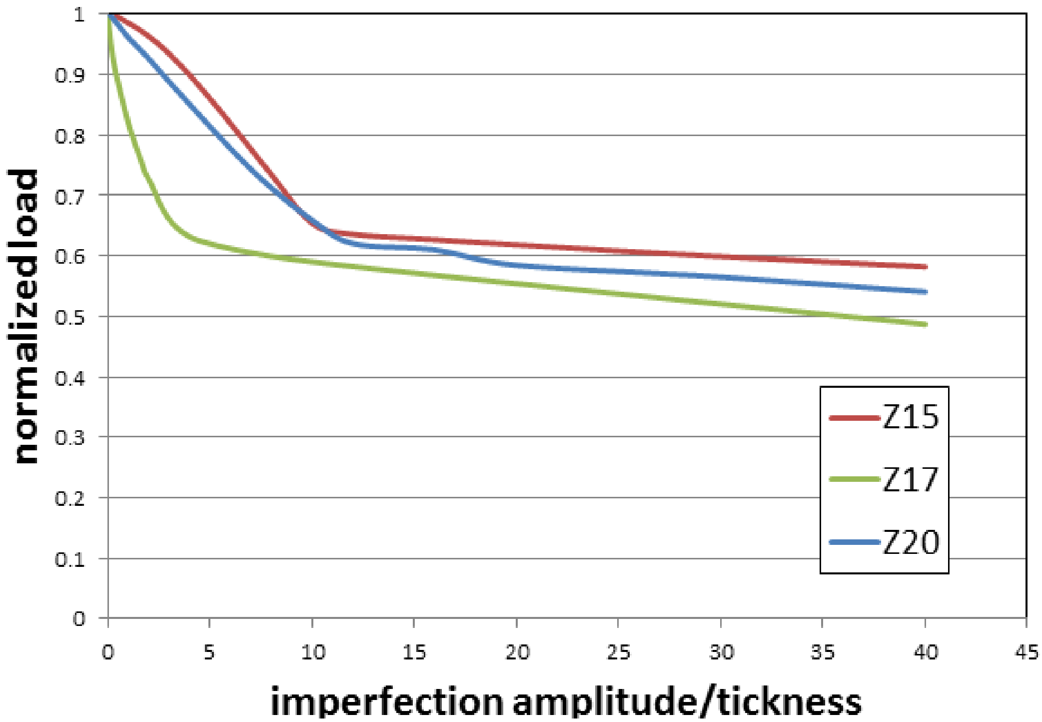
single perturbation load approach applied to **stiffener model**

- SPL on stiffener
- SPL in skin



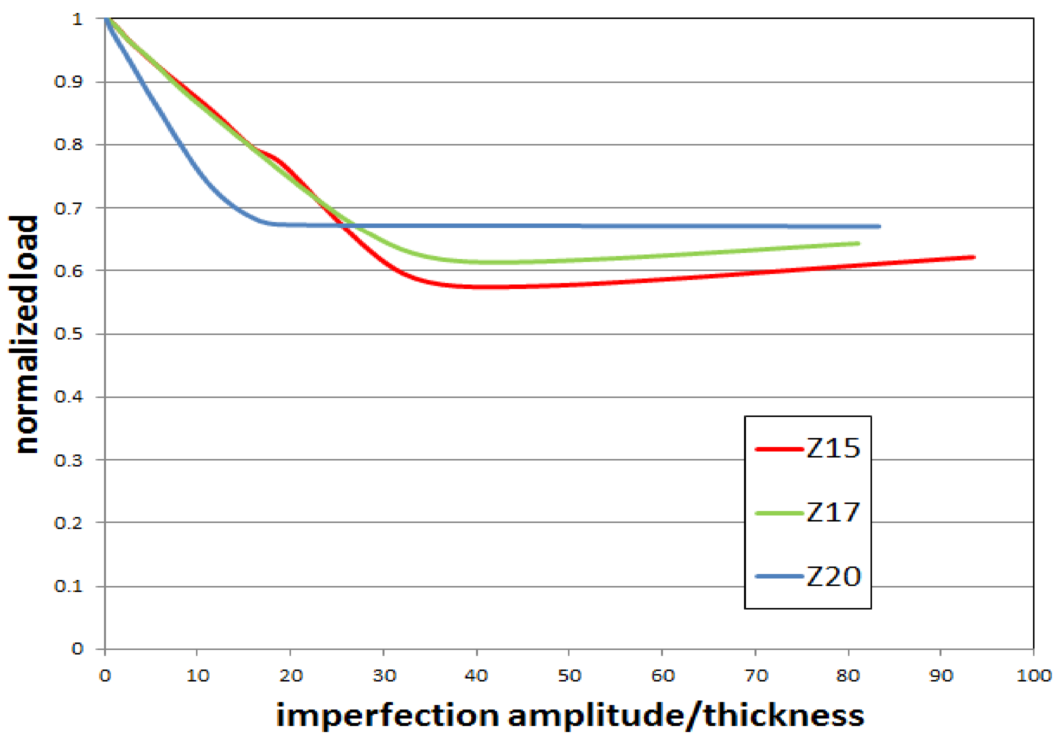
imperfection approach applied to **smeared model – cylinder A**

with averaged knock-down factors from results of three measurements Z15, Z17, Z20



imperfection approach applied to **smeared model – cylinder** B

with averaged knock-down factors from results of three measurements Z15, Z17, Z20



method	cylinder A		cylinder B	
	0 bar	*0.2 bar*	*0 bar*	*0.2 bar*
SPLA	**0.620**	**0.800**	**0.640**	**0.828**
meas. geometric imperfections	**0.621** *(rel dev. 0.29%)*	**0.785** *(rel dev. 1.87%)*	**0.638** *(rel dev. 0.31%)*	**0.804** *(rel dev. 2.89%)*

- here: sufficient correspondence
- k_1 used from single perturbation load approach

$$F_{design} = F_{perfect} \times k_1 \times k_2$$

k_1 considers **geometric imperfection** using deterministic methods

k_2 considers **other imperfections** using stochastic methods

cases considered

(1) geometric imperfection **not included**
applied to the **smeared model to obtain k_2**

(2) geometric imperfection (Z15, Z17, Z20) **included**
applied to the **smeared model for comparison with new KDF**

Monte Carlo simulation based on ABAQUS

buckling considered as probabilistic phenomenon due to distribution of input parameters

scatter of input parameters

- material
- thickness
- geometric imperfection
- load imperfection

Matlab

nonlinear buckling analyses

ABAQUS

analysis results

- results provide distribut. of buckling loads
- lower bound defined with 99% confidence level determines KDF

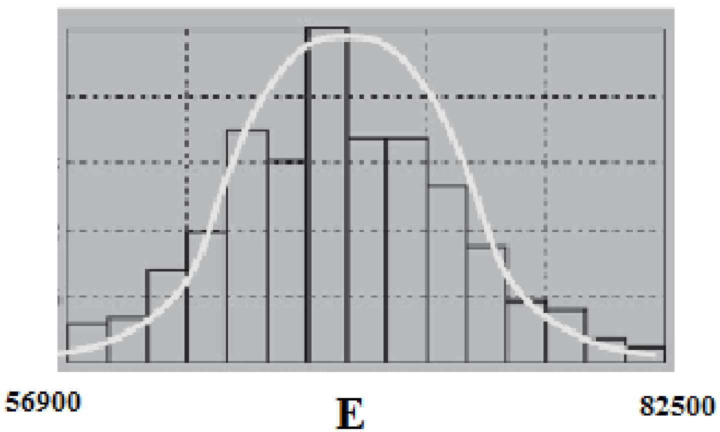
Python & Matlab

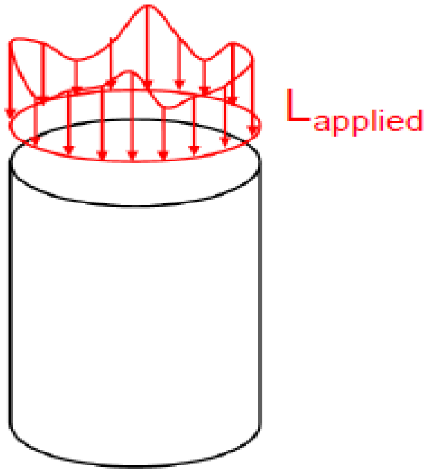
input parameter distribution – **stochastic study**

assumed normal distribution of input parameters (material, thickness skin & stiffener, applied compressive load) with

- a coefficient of variation (CV) = 5%   (measure of dispersion)
    - σ :   standard variation
    - mean μ := initial design / measured value
- number of samples used: 5000
- examples: modulus of elasticity, applied load

$$CV = \frac{\sigma}{\mu}$$

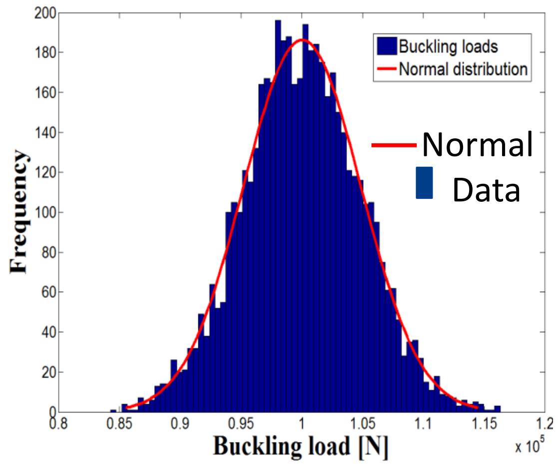




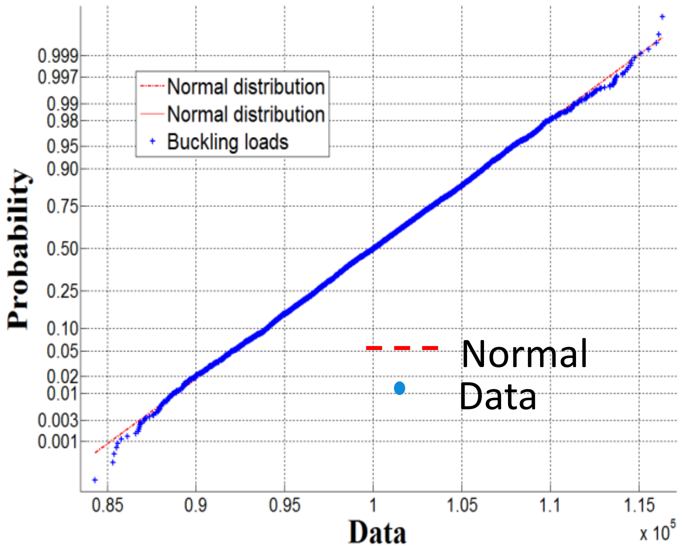
used checks for normal distribution of the input parameter

mean μ = initial design / measured value

(1) histogram



(2) cumulative distribution function (CDF)



(3) Lilliefors test: data accept the normal hypothesis with a 99% confidence level

- CV (coef. of variation) of load imperfection was varied: 3% 5% 10%

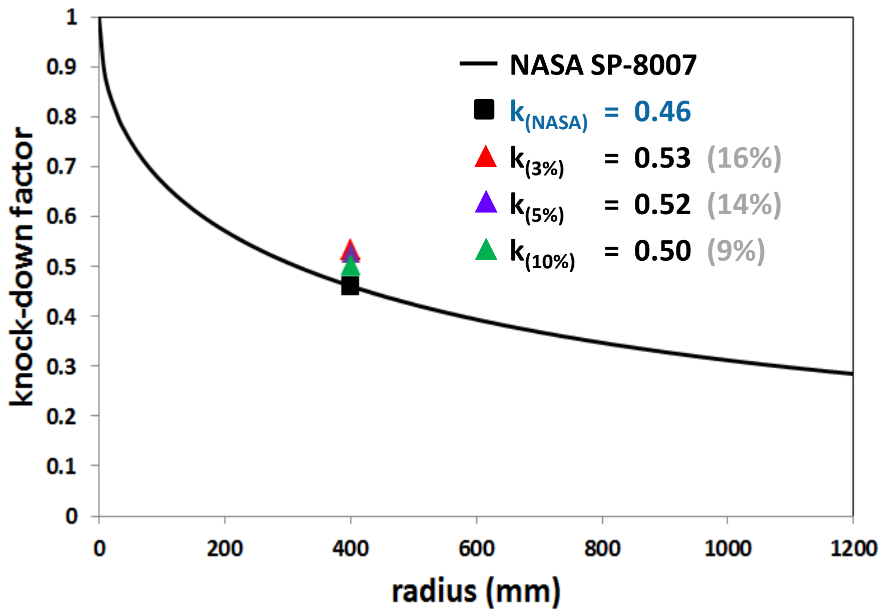
method		cylinder A	cylinder B	
			0 bar	*0.2 bar*
geometric imperfections **not included**	**CV=3%**	**0.86**	**0.85**	**0.89**
	CV=5%	**0.85**	**0.83**	**0.87**
	CV=10%	**0.81**	**0.79**	**0.84**
stochastic with geometric imperfections **included**	**Z15**	**0.70**	**0.61**	**0.79**
	Z17	**0.65**	**0.63**	**0.78**
	Z20	**0.68**	**0.66**	**0.81**

combined knock-down factors – **design values**

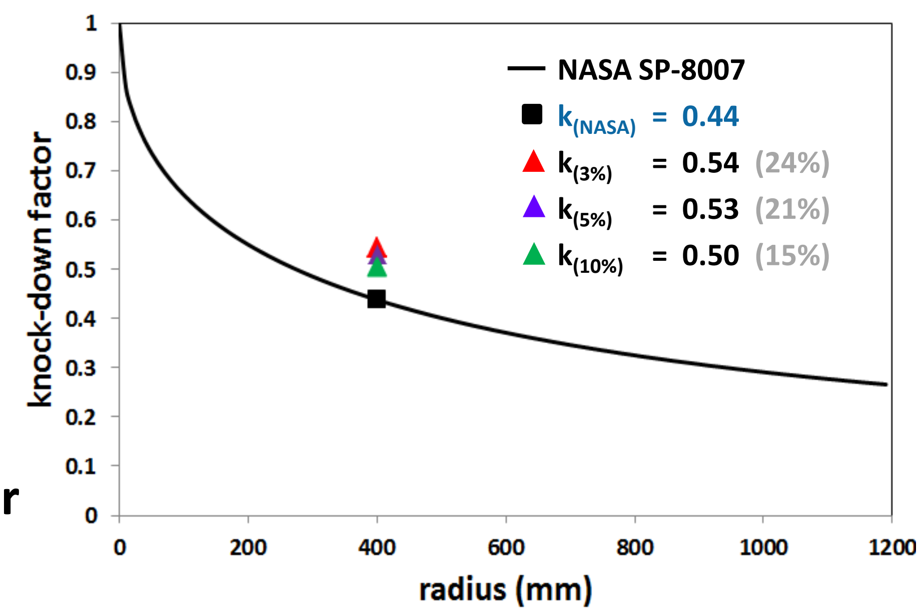
$$F_{design} = F_{perfect} \times k_1 \times k_2$$

method		cylinder A	cylinder B	
			0 bar	*0.2 bar*
$k = k_1 \times k_2$ $k_1 \rightarrow$ geometric imperfect. $k_2 \rightarrow$ other imperfections	**CV=3%** **CV=5%** **CV=10%**	**0.53** **0.52** **0.50**	**0.54** **0.53** **0.50**	**0.74** **0.72** **0.69**
stochastic with geometric imperfections **included**	**Z15** **Z17** **Z20**	**0.70** **0.65** **0.68**	**0.61** **0.63** **0.66**	**0.79** **0.78** **0.81**

combined knock-down factors – **design values**

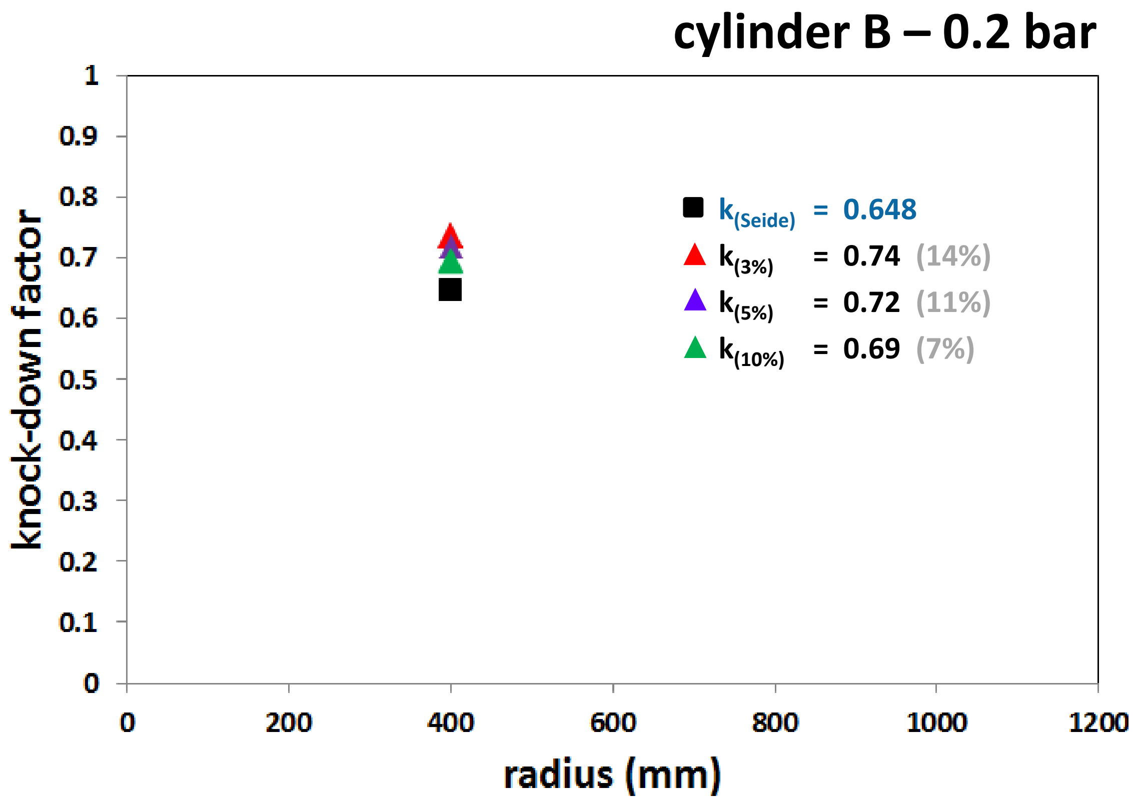


cylinder A – 0 bar



cylinder B – 0 bar

combined knock-down factors – **design values**



summary / conclusions

- **buckling performance** of two stiffened cylinders was analysed
- **smeared model** used
  - considers measured geometric imperfections
  - reduces computational complexity in stochastic MC-based analysis
- **two knock-down factors** derived
  - k_1 deterministic analysis → geometric imperfections
  - k_2 stochastic analysis → other imperfections (load, material,...)
- combined approach is
  - robust and less conservative compared to NASA SP8007
  - more conservative than a pure stochastic approach

DESICOS

New Robust DESign Guideline for Imperfection Sensitive COmposite Launcher Structures

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